The change in the orientation of the evaporation head (horizontal and vertical positions) and also the increase in the temperature of the cooling water to 60°C had an insignificant effect on the amount of the removed power and the temperature within the heater.

## NOTATION

d, $l$, diameter and length of the porous sample, respectively; $T_1$-$T_{10}$, temperatures, measured by the thermocouples 1-10 (Fig. 1); $\Delta T = T_9 - T_3$, radial temperature drop along the wick; Q, supplied power; $Q_{300°}$, $q_{300°}$, $\alpha_{300}$, supplied power, supplied heat-flux density, heat-transfer coefficient at the temperature $T_8 = 300°C$, respectively.

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# THREE-DIMENSIONAL RADIATIVE HEAT-TRANSFER PROBLEM WITH SHADING


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UDC 536.3



The problem of radiative heat transfer between diffuse gray surfaces bounding a closed volume of arbitrary configuration is discussed.


Often in calculations of the heating of airframe structures it is necessary to solve problems of radiative heat transfer between the surfaces of various structural elements forming the interior compartments of an aircraft. In many cases the entire bounding surface is nonconvex and has such a complex configuration as to present serious difficulties in applying the zonal method.

For situations in which one of the dimensions of such a bounded volume is much greater than all the rest, we have proposed [1] a method for solving the planar radiative heat-transfer problem with allowance for shading and have demonstrated the substantial influence of this factor on the distribution of $q_{inc}$ over the surface of compartments of real structures. In the present study we elaborate the method of analysis of radiative heat transfer with allowance for shading in the three-dimensional case.

We consider the problem of radiative heat transfer between diffuse gray surfaces bounding a closed volume of arbitrary configuration. An open volume can be closed by the addition of a fictitious surface with $\varepsilon = 1$ and $T = (q_\infty/\sigma)^{1/4}$, where $q_\infty$ is the dissipated heat flux from the surrounding medium.

We assume that the bounding surface comprises N plane faces having the shape of a convex rectangle. These are actually the kind of surfaces that occur in the majority of real problems, and any continuous surface can always be approximated with sufficient accuracy by a system of plane faces. The temperature and emissivity distributions over each face are variable.

The radiative heat transfer in such a region is described by a system of Fredholm integral equations of the second kind in the incident flux density:

$$q_{\text{inc}}(p_i) = \sum_{j=1}^{N} \int_{F_j} \{\sigma\varepsilon(p_j) T^4(p_j) + [1 - \varepsilon(p_j)]\, q_{\text{inc}}(p_j)\} K(p_i, p_j)\, dF_j, \quad i = 1, 2, \ldots, N, \tag{1}$$

where $K(p_i, p_j)$ is a function of the angular coefficients:

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$$K(p_i,\ p_j) = K(p_j,\ p_i) = \begin{cases} \dfrac{\cos\varphi_1(p_i,\ p_j)\cos\varphi_2(p_j,\ p_i)}{\pi R^2(p_i,\ p_j)} & \text{if} \quad \varkappa=0, \\ 0 & \text{if} \quad \varkappa=1, \end{cases} \tag{2}$$

$\varphi_1$ and $\varphi_2$ are the angles between the normals to the i-th and j-th faces and the line segment of length $R(p_i,\ p_j)$ joining points $p_i$ and $p_j$, and $\varkappa$ is the visibility parameter: The point $p_i$ "sees" the point $p_j$ for $\varkappa=0$ and does not see it for $\varkappa=1$.

Points $p_i$ and $p_j$ are mutually visible if the angles between the normals to the surfaces on which the points are situated and the segment $[p_i,\ p_j]$ have absolute values less than $\pi/2$, i.e., if

$$\cos\varphi_1(p_i,\ p_j) > 0, \quad \cos\varphi_2(p_j,\ p_i) > 0 \tag{3}$$

and the segment $[p_i, p_j]$ does not intersect any faces of the surface other than the i-th and j-th faces. The latter conditions holds if either the segment does not intersect the planes through these faces, i.e.,

$$|2t_k - 1| > 1, \quad k = 1,\ 2,\ \dots,\ N,\ k \neq i,\ j, \tag{4}$$

where

$$t_k = \frac{A_k x_i + B_k y_i + C_k z_i + D_k}{A_k(x_j - x_i) + B_k(y_j - y_i) + C_k(z_j - z_i)},$$

and $A_k$, $B_k$, $C_k$, $D_k$ are the coefficients of the equation for the plane passing through the k-th face, which are expressed in terms of the coordinates of three corners of the face:

$$\begin{vmatrix} x & y & z & 1 \\ x_{1,k} & y_{1,k} & z_{1,k} & 1 \\ x_{2,k} & y_{2,k} & z_{2,k} & 1 \\ x_{3,k} & y_{3,k} & z_{3,k} & 1 \end{vmatrix} = 0,$$

or the point of intersection $p_k$ does not belong to the k-th face, as is indicated by failure of at least one of the conditions

$$t_{k,s} > 1, \quad s = 1,\ 2,\ 3,\ 4, \tag{5}$$

where

$$t_{k,s} = \frac{(x'_s - x'_c)(y'_s - y'_c) - (x'_s - x'_{s+1})(y'_s - y'_{s+1})}{(x'_k - x'_c)(y'_s - y'_{s+1}) - (x'_s - x'_{s+1})(y'_k - y'_c)}, \quad s = 1,\ 2,\ 3,\ 4,$$

$$x'_{4+1} = x'_1, \quad y'_{4+1} = y'_1,$$

$$x'_c = \frac{1}{4}\sum_{s=1}^{4} x'_s, \quad y'_c = \frac{1}{4}\sum_{s=1}^{4} y'_s,$$

$x'_s$, $y'_s$ (s = 1, 2, 3, 4), $x'_k$, $y'_k$ are the coordinates of the corners of the k-th face and the point of intersection in the local coordinate system $0x'y'$.

We now proceed with the solution of the system of equations (1). On each face we overlay a variable-step computing grid with boundary points situated at the faces (see Fig. 1). We denote the number of grid nodes on the j-th face by $M_j = SR$, and the number of mesh cell by $L_j = (S-1)(R-1)$, where S and R are the numbers of nodes in the direction of the axes $0x'$ and $0y'$, respectively. We enumerate the cells in such a way that

$$l = s + (r-1)(S-1), \quad s=1,\ 2,\ \dots,\ S,\ r=1,\ 2,\ \dots,\ R,$$

and the order numbers of the nodes comprising the corners of these cells are, respectively,

$$\begin{aligned} m_{l,1} &= s + (r-1)S, & & \\ m_{l,2} &= s + rS, & s &= 1,\ 2,\ \dots,\ S, \\ m_{l,3} &= s + 1 + rS, & r &= 1,\ 2,\ \dots,\ R, \\ m_{l,4} &= s + 1 + (r-1)S. & & \end{aligned}$$

From the system of equations (1) we obtain the following expression for the incident flux density at the nodes:

$$q_{\text{inc}\,i,m} = \sum_{j=1}^{N} \sum_{l=1}^{L_j} \int_{F_{j,l}} \{\sigma\varepsilon(p_j) T^4(p_j) + [1-\varepsilon(p_j)]\, q_{\text{inc}}(p_j)\} K(p_{i,m}, p_j)\, dF_j, \tag{6}$$

$$i = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, M_i.$$

We denote

$$f(p) = \sigma\varepsilon(p) T^4(p) + [1-\varepsilon(p)]\, q_{\text{inc}}(p) \tag{7}$$

and approximate the function (p) in the $l$-th cell of the j-th face by the linear function

$$\Phi(x', y') = a_{1,l} + a_{2,l} x' + a_{3,l} y' + a_{4,l} x' y'. \tag{8}$$

The coefficients $a_{t,l}$ of the approximating function are determined from the condition that it coincide with f(p) at the corners of the cell:

$$a_{1,l} + a_{2,l} x'_{m_{l,\tau}} + a_{3,l} y'_{m_{l,\tau}} + a_{4,l} x'_{m_{l,\tau}} y'_{m_{l,\tau}} = f_{j,m_{l,\tau}}, \tag{9}$$

$$\tau = 1, 2, 3, 4.$$

Solving (9), we obtain

$$a_{t,l} = \sum_{\tau=1}^{4} \frac{A_{t,m_{l,\tau}}}{\Delta_{j,l}} f_{j,m_{l,\tau}}, \tag{10}$$

where $\Delta_{j,l}$ is the determinant of the system of equations (9) and $A_{t,m_{l,\tau}}$ denotes the corresponding signed minors.

Taking (7), (8), and (10) into account, we express the integrals in (6) in terms of the values of the function f(p) at the nodes:

$$\int_{F_{j,l}} f(p_j) K(p_{i,m}, p_j)\, dF_j = \sum_{\tau=1}^{4} B_{j,m_{l,\tau}} f_{j,m_{l,\tau}}, \tag{11}$$

where

$$B_{j,m_{l,\tau}} = \frac{1}{\Delta_{j,l}} \Big\{ A_{1,m_{l,\tau}} \int_{F_{j,l}} K(p_{i,m}, p_j)\, dF_j + A_{2,m_{l,\tau}} \times$$
$$\times \int_{F_{j,l}} x' K(p_{i,m}, p_j)\, dF_j + A_{3,m_{l,\tau}} \int_{F_{j,l}} y' K(p_{i,m}, p_j)\, dF_j +$$
$$+ A_{4,m_{l,\tau}} \int_{F_{j,l}} x' y' K(p_{i,m}, p_j)\, dF_j \Big\}. \tag{12}$$

The coefficients $B_{j,m_{l,\tau}}$ depend only on the form of the surface and its partitioning. The integrals entering into (12) are computed numerically according to Korobov's algorithm for the computation of multiple integrals whose integrands do not contain singularities of order higher then $x^{-1/2}y^{-1/2}$. Although the integrands in (12) have higher-order singularities in the cells adjacent to points $p_{i,m}$ situated along the edges and at the corners, the corresponding integrals are completely determined by finite functions of the points $p_i$, and as $p_i$ tends to $p_{i,m}$ the value of the integral tends asymptotically to its value at the point $p_{i,m}$. Accordingly, the

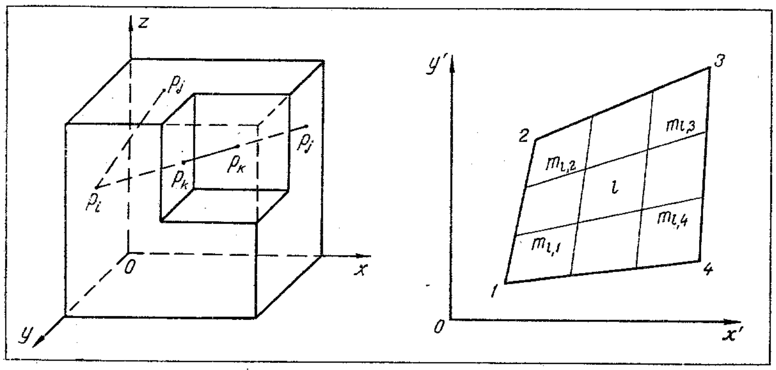


Fig. 1. Computing scheme.

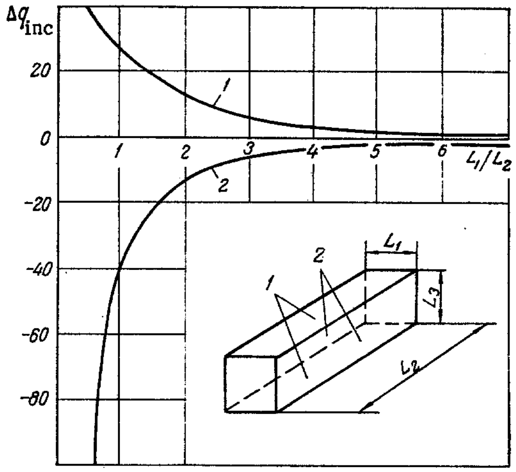


Fig. 2. Errors of determination of $q_{inc}$, %, according to a plane computational scheme.

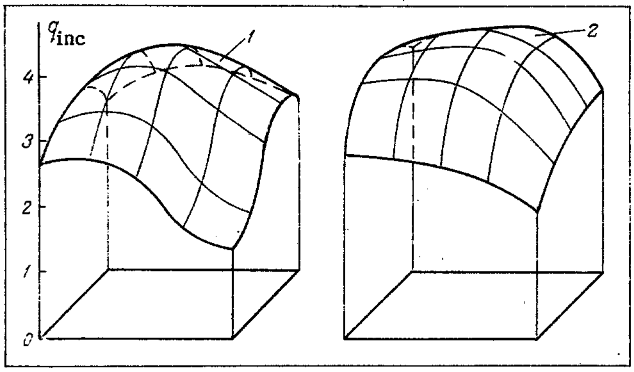


Fig. 3. Distribution of incident heat-flux density $q_{inc}$ (kW/m$^2$) over lower face of a cube: 1) with shading; 2) without shading.

computation of the integral for the point $p_{i,m}$ with a singular integrand is replaced by computation of the integral for a point $p_{i,m}+\delta$ in the neighborhood of $p_{i,m}$ such that the integrand is bounded at that point. For points $p_{i,m}$ situated along an edge, $p_{i,m}+\delta$ is taken on the perpendicular to the edge, and for such points situated at a corner the new point is taken on the median.

Substituting (11) into (6), grouping terms by nodes, and taking (7) into account, we obtain a system of linear algebraic equations in the incident heat-flux densities at the nodes:

$$q_{inc\ i,m} = \sum_{j=1}^{N} \sum_{n=1}^{M_j} \{\sigma\varepsilon_{j,n} T_{j,n}^4 + (1-\varepsilon_{j,n}) q_{inc\ j,n}\} H_{i,m,j,n}, \tag{13}$$

$$i = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, M_i,$$

where H is the matrix, analogous to the matrix of local angular coefficients, with elements

$$H_{i,m,j,n} = \sum_{m_{l,\tau}=n} B_{j,m_{l,\tau}}. \tag{14}$$

As in the case of the matrix of local angular coefficients, this matrix satisfies the closure conditions

$$\sum_{j=1}^{N} \sum_{n=1}^{M_j} H_{i,m,j,n} = D_{i,m} = 1, \tag{15}$$

$$i = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, M_i.$$

Inasmuch as the satisfaction of conditions (15) is essential for convergence of the iterative solution of the system (13) and as certain violations of those conditions are possible in computer calculations as a result of computational errors, before the system can be solved the elements of the matrix H must be normalized:

$$H_{i,m,j,n} = \frac{H_{i,m,j,n}}{D_{i,m}},$$

$$i,\ j = 1,\ 2,\ \ldots,\ N,\quad m,\ n = 1,\ 2,\ \ldots,\ M_i\ (M_j).$$

The system (13) is solved iteratively in order to include the temperature dependence of the emissivity in conductive and radiative heat-transfer problems. To speed up the convergence of the iterative process we invoke the principle of conservation of energy in radiative heat transfer within a closed volume:

$$\oint_F \varepsilon(p)\, q_{\text{inc}}(p)\, dF = \oint_F \sigma\varepsilon(p)\, T^4(p)\, dF. \tag{16}$$

Here we use as the initial data for the k-th iteration, rather than the values of $q_{\text{inc}\ i,m}^{(k-1)}$, the quantities

$$q_{\text{inc}\, i,m} = q_{\text{inc}\, i,m}^{(k-1)} + \theta, \tag{17}$$

where

$$i = 1,\ 2,\ \ldots,\ N,\quad m = 1,\ 2,\ \ldots,\ M_i,$$

$$\theta = \frac{\sum\limits_{j=1}^{N}\sum\limits_{n=1}^{M_j} C_{j,n}\left(\sigma\varepsilon_{j,n}T_{j,n}^4 - \varepsilon_{j,n}q_{\text{inc}\, j,n}^{(k-1)}\right)}{\sum\limits_{j=1}^{N}\sum\limits_{n=1}^{M_j} C_{j,n}\varepsilon_{j,n}},$$

and $C_{j,n}$ denotes the coefficients of the representation of the integrals in (16) in terms of the values of the integrand at the nodes when approximated by expression (8) in the cell.

We have developed a FORTRAN program in accordance with the proposed method. We have used the program to perform calculations of the radiative heat transfer in a rectangular parallelepiped with square contours of various lengths. The temperature of the ends and vertical faces is assumed to be constant and equal to 300°K, and the temperature of the horizontal faces is assumed to be a constant 800°K. The emissivity of all the faces is considered to be identical and equal to 0.5. The calculations are carried out according to a plane scheme for the same contour. Figure 2 gives the corresponding deviations of the values of $q_{\text{inc}}$ at the center of the sides of the contour from the values of $q_{\text{inc}}$ at the center of the horizontal and vertical faces of the parallelepiped. It follows from the given data that the errors of computation of $q_{\text{inc}}$ according to the plane scheme are substantial for elongations $L_2/L_1 < 3$.

We have carried out a computation with allowance for shading in the case of a cube with its first quarter excised (see Fig. 1). The temperature of the upper face is assumed to be 800°K, and the temperature of all other faces 300°K. All faces have the same emissivity, equal to 0.5. Figure 3 gives the resulting distribution of $q_{\text{inc}}$ over the surface of the lower face. Also given in Fig. 3 is the distribution of $q_{\text{inc}}$ for the case in which shading is absent, i.e., for the whole cube with the same region at a temperature of 800°K on the upper face as when shading is taken into account.

## NOTATION

x, y, z, coordinates; x', y', coordinates of local system; T, temperature; $q_\infty$, heat flux from surrounding medium; $q_{\text{inc}}$, incident heat flux; $\sigma$, Stefan–Boltzmann constant; $\varepsilon$, emissivity; $K(p_i, p_j)$, function of angular coefficients; $\varphi$, angle; $p_i$, $p_j$, surface points; N, number of faces; $F_j$, area of j-th face; $M_j$, $L_j$, number of nodes and cells on j-th face; $\varkappa$, visibility parameter; $t_k$, $t_{k,s}$, parameters.

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